Climatic variability on $10^3$- to $10^4$-year timescales and an associated inter-hemispheric seesaw pattern during the last glacial period has been documented in palaeoclimatic records. However, their frequencies in older glacial periods and relationships with mean climatic state are still unclear. We investigate the long-term characteristics of such variability using a new 700,000-year (700-kyr) ice core record from Dome Fuji, East Antarctica. Combining with another long Antarctic record, we identify $10^3$- to $10^4$-year warming events over the past eight glacial cycles. They are most frequent and abundant when Antarctic temperature is slightly below average, equivalent to an intermediate climatic state during glacial periods. The relationship between event frequency and climatic state is stable across Mid-Brunhes Event ~430 kyr ago, after which the amplitude of glacial-interglacial cycles increases. We also observe events with long periodicity (>10^4 years) and large magnitude during early stages and terminations of glacial periods, when Northern Hemisphere summer insolation is highly variable, suggesting a role of orbital forcing in the inter-hemispheric seesaw. With a fully coupled climate model, we conducted a number of 1,000-yr-long freshwater hosing experiments under glacial and interglacial climate conditions. Under a glacial condition, even with moderate fresh water anomaly, the model demonstrates a complete cycle of inter-hemispheric seesaw in temperature and precipitation. It reproduces abrupt changes in Northern North Atlantic region and gradual changes in Antarctica, quantitatively consistent with available data. On the other hand, under an interglacial climate, much larger water anomaly is needed to simulate such changes. Together with the ice core data, this suggests that the prerequisite for climate instability during the late Pleistocene is the combination of high frequency (not necessarily large amount) of freshwater into Northern North Atlantic from continental ice sheets and high sensitivity of climate to freshwater anomaly.